...GOING UP...

AN ESSAY ON "HYPERBOLIC ELECTRONS AND THE FIFTH FORCE" OF Dr. RANDELL MILLS'S "GUT_CP"

Ben Jones, MPhys., MSc.

References to "GUT_CP" are to the July 2010 edition.

As a prerequisite to understanding this paper, readers should have some familiarity, although not detailed understanding of, Dr. Randell Mills's GUT_CP [1]. Also, readers unfamiliar with the concepts of *"tractrix", "pseudosphere", "hyperboloid",* and *"Gaussian Curvature"* are advised to look them up (*e.g. Wikipedia*) to assist in understanding this paper.

Let us suppose that a large number of the particles and bodies in the Universe are made up of either 'Orbitspheres' or 'Orbitsphere-like structures'. An 'Orbitsphere' is defined as a two-dimensional surface made up of a huge number of filamentary closed curves along which mass and charge circulate. These filamentary curves may have many intersections with each other as they traverse the surface, or they may not intersect with each other at all.

Dr. Mills introduces the concept of the Orbitsphere for an electron in a hydrogen atom in Chapters 1 and 2 of GUT_CP. In Chapter 3 he discusses the Free Electron using related concepts.

In Chapter 35 he introduces the concept of the Hyperbolic Electron. What is most special about the Hyperbolic Electron is that it has negative gravitational mass, and so it accelerates upwards in a gravitational field.

1.1 POSITIVE AND NEGATIVE GRAVITATIONAL MASS - THE KEY CONCEPTS

On page 1612/1832[1] of GUT_CP, Mills quotes the formula for the Schwarzchild metric,

$$d\tau^2 = \left(1 - \frac{2Gm_0}{c^2 r}\right)dt^2 - \frac{1}{c^2}\left[\left(1 - \frac{2Gm_0}{c^2 r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]. \qquad (1)$$

The meaning of the variable r in this equation, in the context of GUT_CP, is somewhat different from its original meaning in Schwarzchild's solution to the field equations of General Relativity. For an Orbitsphere or Orbitsphere-like structure, the variable r in equation (1) refers to the *radius of Gaussian curvature on the velocity-transformed surface.*

[1] In this paper we reference pages of GUT_CP according to the page number in the DjVu viewer, which is different from the page number that appears on printed pages of GUT_CP.

What we mean by the *velocity-transformed surface* is the following:

For the bound electron Orbitsphere, the velocity of an element of mass (charge) is constant for every point on a particular filamentary circle, and also this velocity value is the same for all filamentary circles. We transform the Orbitsphere by changing each filamentary circle's radius to a value equal to a constant multiplied by the velocity of mass (charge) flow in that filamentary circle. For the bound electron Orbitsphere, the velocity is constant for all filamentary circles, and so, under this transformation, the filamentary circles all are multiplied by the same factor and the resulting transformed surface looks the same as the original Orbitsphere, apart from being expanded or contracted.

For an Orbitsphere or Orbitsphere-like structure where the mass (charge) flow velocity varies from one filamentary circle to the next filamentary circle, the different filamentary circles are transformed into circles with different radii. For each filamentary circle, the transformed circle is coplanar and concentric with the original filamentary circle. The surface formed by the transformed structure of filamentary circles is the *velocity-transformed surface.*

Figures 1 and 2 illustrate the transformation starting with an Orbitsphere-like structure consisting of a cylindrical arrangement of Filamentary Circles.

In this present paper, we only consider Filamentary Closed Curves which are in fact circles.

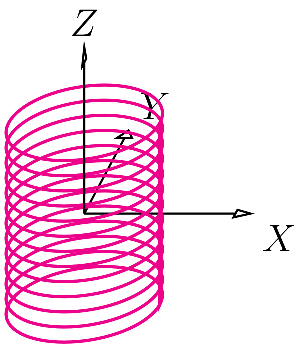

Figure 1: *An cylindrical Orbitsphere-like structure. The Filamentary Circles are in planes parallel to the xy-plane. The velocity of mass (charge) flow around each circle is given by* $v = a + bz^2$ *where* z *is the z-coordinate of the Filamentary Circle.*

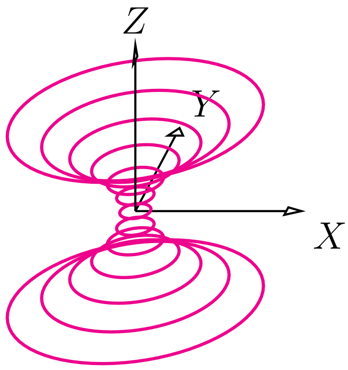

Figure 2: *The* "velocity-transformed surface" *of the Orbitsphere-like structure of Figure 1.*

Next we state the following postulate:

POSTULATE 1:
If the Gaussian Curvature of the *velocity-transformed surface* for an Orbitsphere-like particle is positive, then the gravitational mass is positive.
If the Gaussian Curvature of the *velocity-transformed surface* for an Orbitsphere-like particle is NEGATIVE, then the gravitational mass is NEGATIVE.

In chapter 35 of GUT_CP and the chapters leading up thereto, Mills presents arguments for the above postulate based upon his theories for Creation of Matter from Energy / Pair Production / Spacetime Expansion and Contraction / Reference Frames. In the context of this paper, we take Postulate 1 as an irreducible postulate.

1.2 THE HYPERBOLIC ELECTRON

One of the triumphs of Mills *et alia* appears to be their report on the successful generation of "Hyperbolic Electrons" which have negative gravitational mass.

The Hyperbolic Electron is an Orbitsphere-like structure which forms a spherical surface. It is like the Orbitsphere for the bound electron in that it is spherical, however the structure of Filamentary Circles is different. The Filamentary Circles are exactly like parallels of latitude on a globe. There is one equatorial Great Circle, and there are other filamentary circles in planes parallel to the equatorial plane, and these other filamentary circles get progressively smaller as one goes further from the equatorial plane.

The formula for the mass density on the surface of the Hyperbolic Electron is given by equation 35.72 on page 1623/1832 of GUT_CP:

$$\sigma_m(r,\theta,\phi) = \frac{m_e}{\frac{8}{3}\pi r_0^2} \sin\theta \quad \delta(r - r_0). \tag{2}$$

Another, more simple, way of writing this equation is

$$\sigma_m = A \sin\theta \tag{3}$$

in which A is a constant. Thus, the mass density varies as the sine of the polar angle θ. The charge density varies similarly.

The velocity of mass (charge) flow on each Filamentary Circle is given by equation 35.75 on page 1623/1832 of GUT_CP:

$$\mathbf{v}(r,\theta,\phi,t) = \frac{\hbar}{m_e r_0 \sin\theta}\delta(r-r_0)\mathbf{i}_\phi. \tag{4}$$

The same information is provided by stating

$$v = \frac{A'}{\sin\theta} \tag{5}$$

in which A' is a constant, and by keeping in mind the geometric picture of the velocity being along "parallels of latitude" on the spherical surface of the Hyperbolic Electron.

Figure 3 shows the XZ-profile of the Hyperbolic Electron and also shows a cross-section of the velocity-transformed surface, with the velocity given by equation (5).

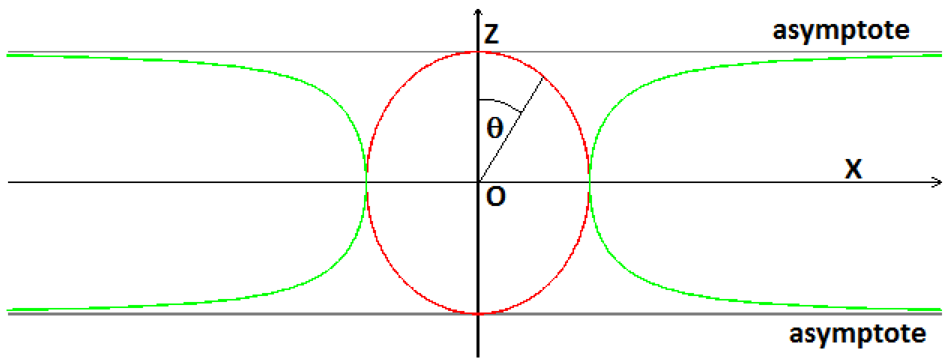

Figure 3: *Red circle is a slice through the spherical surface of the Hyperbolic Electron, the Green curves are the XZ cross section of the velocity-transformed surface. The velocity-transformed surface can be made by revolving the Green curves around the Z-axis.*

The velocity transformed surface whose XZ-planar-cross-section is shown in Figure 3 is such that every point on the surface is *saddle-like*[2] rather than *dome-like*. Therefore, the

[2]We make a somewhat non-rigorous definition here of *dome-like* versus *saddle-like*. Consider a point P on a 2D surface in 3D space. Let T be the tangent plane at P. Then consider a small region of the surface, such that all points of the region are within a distance δ of P. Denote the locus of these points by R. If all points of R are to ONE SIDE of T, then the surface is *dome-like* at P. If some of the points of R are on one side of T and some are on the other side, then the surface is *saddle-like* at P.

Gaussian curvature is negative at every point, and so, if Postulate 1 above is correct, the particle will have negative gravitational mass and, once created, it will be "Going Up" - or, at least, it will experience a force opposite to the ordinary gravitational force.

On page 1625/1832 of GUT_CP, Mills states that the Hyperbolic Electron has

"a velocity function on the surface whose magnitude approaches the limit of light speed at opposite poles (Eq. (35.75))..."

However, equation 35.75 is quoted above (equation (4)) and in fact it shows that at the opposite poles the velocity approaches infinity. It is possible that Mills makes the quoted statement above in anticipation of a more advanced analysis in which the velocity at the poles approaches c. Mills might be wanting to apply the postulate that "*matter or energy cannot travel faster than than the speed of light, c*" to the filamentary circles which make up Orbitspheres and Orbitsphere-like structures.

On pages 1613/1832, 1624/1832, 1635/1832, and 1637/1832 of GUT_CP, Mills refers to the velocity surface of the Hyperbolic Electron as a hyperboloid. However, equation (5) does not technically actually describe a hyperboloid. The surface formed by revolving the green curves of Figure 3 is in many ways similar to a hyperboloid of one sheet, however it asymptotically approaches two parallel planes rather than a double cone. A hyperboloid of one sheet asymptotically approaches a double cone. Again, the fact that it is not an exact hyperboloid may have been deliberately neglected by Mills in anticipation of more advanced analysis.

The next question we consider is *"How do Mills* et alia *generate Hyperbolic Electrons?"*. To try to understand the experimental apparatus described (GUT_CP Chapter 35) by Mills *et alia* for generating hyperbolic electrons by impacting free electrons into neutral atoms, we next look at Mills's theory of the Free Electron.

1.3 THE FREE ELECTRON

In standard Quantum Mechanics, the free electron is often thought of as a point particle having a probability distribution. In contrast, in Dr. Mills's GUT_CP, the free electron is considered to have an actual structure. Chapter 3 of GUT_CP describes the Free Electron as an Orbitsphere-like structure. The main concepts are

- The free electron is an extended distribution of charge (mass) about a planar disc. The disc is aligned perpendicular to the translational velocity vector of the electron.
- The charge (mass) density, that is, charge or mass per unit area of the planar disc, is greatest at the centre of the disc, and decreases as one moves radially outwards.
- The charge (mass) density at a radius ρ from the centre is given by

$$\sigma_{charge,mass} = constant \times \sqrt{\rho_0^2 - \rho^2} \tag{6}$$

in which ρ_0 is the radius of extent of the disc. GUT_CP Equation 35.47 for the charge density is

$$\sigma_e(\rho, \phi, z) = \frac{e}{\frac{2}{3}\pi\rho_0^3}\sqrt{\rho_0^2 - \rho^2} \tag{7}$$

and GUT_CP Equation 35.46 for the mass density is

$$\sigma_m(\rho, \phi, z) = \frac{m_e}{\frac{2}{3}\pi\rho_0^3}\sqrt{\rho_0^2 - \rho^2} \tag{8}$$

See Appendix A for the verification that this formula for the mass per unit area yields, over the planar disc, a total mass of m_e = electron mass.

- The planar disc of charge (mass) is made up of filamentary circles of flowing charge (mass). These filamentary circles are all centred on the centre of the disc. The velocity along a filamentary circle is proportional to the radius from the centre, ρ. To see the charge (mass) flowing with this velocity, it is necessary to make the observation in a frame moving at the translational velocity of the electron, in which frame the disc has no translational motion. The velocity about a filamentary circle is given by

$$\mathbf{v}(\rho, \phi, z, t) = \left[\frac{5}{2}\rho\frac{\hbar}{m_e\rho_0^2}\mathbf{i}_\phi\right] \tag{9}$$

(Equations 35.33 and 3.37 of GUT_CP)

- the de Broglie wavelength is related to ρ_0 via

$$2\pi\rho_0 = \lambda_{de\ Broglie} \tag{10}$$

(Equations 35.44 and 3.45 of GUT_CP). This is equivalent to saying that the circumference of the disc is equal to the de Broglie wavelength.

Figure 4 shows the mass per unit area as a function of ρ for the propagating free electron. It is similar to Figure 3.2B of GUT_CP.

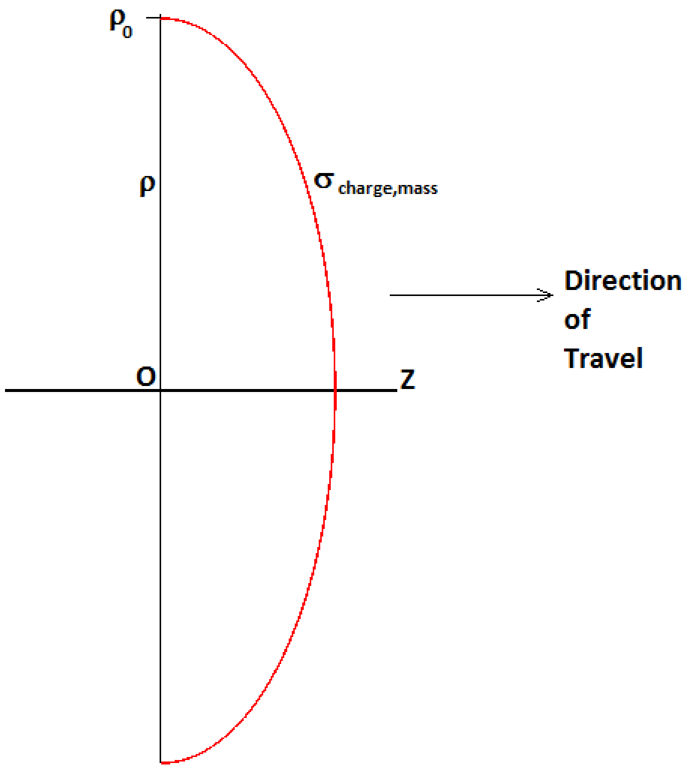

Figure 4: *Charge (mass) density plotted, using equation* (6)*, as a function of the radius from the centre of the planar disc. The planar disc is the shape assumed by the propagating free electron.*

ALTERNATIVE VIEW: ELLIPSOIDAL FREE ELECTRONS

Note that equation (6) is also the equation for the upper half of an ellipsoid, and that the Red curve of Figure 4 depicts the cross section of a half-ellipsoid. To make the ellipsoid, one can combine the Red curve with its reflection through a plane perpendicular to the Z-axis and intersecting the origin to form an ellipse, and then revolve this ellipse about the Z-axis. Rather than describing the electron as a planar disc of zero thickness and varying mass, it might be possible to describe it as a *"pill shaped"* ellipsoid of uniform mass (charge) per unit volume.

RANGE OF RADII OF EXTENT

The de Broglie wavelength is equal to the circumference about the planar-disc shaped Free Electron. By generating free electrons of a variety of speeds, one is able to generate a variety of de Broglie wavelengths and hence a variety values for the radius of extent, ρ_0. Figure 5 depicts the variation ρ_0 and de Broglie wavelength with propagational speed.

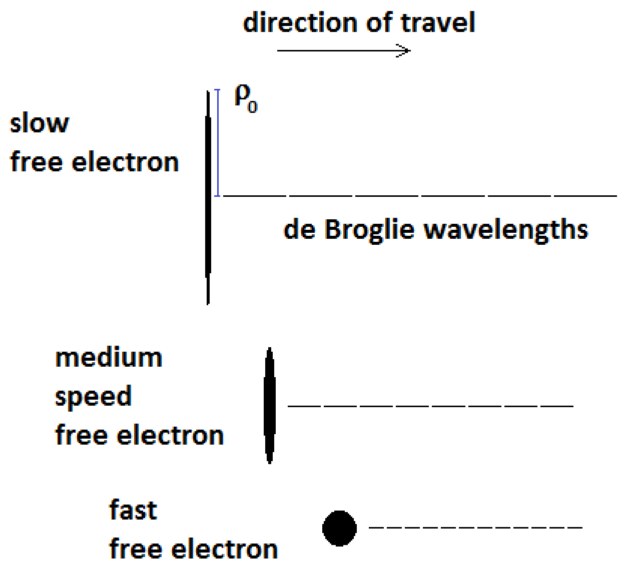

Figure 5: *Sketch illustrating how a slow electron has a large radius of extent ρ_0 and a large de Broglie wavelength, whereas a fast electron has a small radius of extent ρ_0 and a small de Broglie wavelength. The dashed lines are drawn so that each dash corresponds to one de Broglie wavelength. Even though the electrons are supposed to be planar discs, an element of "fatness" along the horizontal (propagation) direction is shown to illustrate how the mass (charge) varies as a function of distance from the centre of the planar disc. Because the three electrons all contain the same mass (charge), the electrons with smaller ρ_0 are depicted as fatter near their centres.*

1.4 GENERATION OF HYPERBOLIC ELECTRONS

In Table 35.1 of GUT_CP, Mills gives values of parameters used for the production of Hyperbolic Electrons in his experiment involving crossing an electron beam with an atomic beam. The Table has columns for the following parameters

- Peak #
- Theoretical Hyperbolic Electron Radius
- Theoretical Velocity
- Theoretical Threshold Kinetic Energy
- Quantum Numbers

At first glance, the Table appears to suggest that when the free electrons have the *"Theoretical Threshold Kinetic Energy"*, then what happens is a free electron hits an inert-gas atom of the atomic beam, and its translational velocity is effectively reduced to zero by the impact, and that the intial kinetic energy is used to cause an internal transition within the structure of the electron. The planar disc transforms into a spherical-shaped Hyperbolic Electron, and the energy required for it to make this transition is equal to the *"Theoretical Threshold Kinetic Energy"*.

However, upon further study of this section of GUT_CP, it has become apparent to the present author that this energetic argument does NOT actually describe the mechanism for Hyperbolic Electron formation. Rather, it appears to be the case that the critical factor for Hyperbolization of a free electron is that the free electron must have a radius ρ_0 equal to a Hyperbolic Electron radius. Hyperbolic Electrons cannot have just any radius - the allowed radii are those shown in Table 35.1 under the column *"Theoretical Hyperbolic Electron Radius"*. When the free electron undergoes Hyperbolization, its shape transforms such that the sphere-shaped Hyperbolic Electron has a radius r_0 equal to what the planar disc radius ρ_0 of the free electron was.

We can test this idea using the given data for Peak #1 in Table 35.1. We suppose we have, initially, a free electron of radius

$$\rho_0 = 0.5670 a_0 \tag{11}$$

in which a_0 = Bohr Radius = 5.29177 * 10^{-11} m [3].

Since we have, from equation (10)

$$2\pi\rho_0 = \lambda_{de\ Broglie} \tag{12}$$

we can use the de Broglie relationship

$$p = m_e v_z = \frac{h}{\lambda_{de\ Broglie}} \tag{13}$$

and obtain

$$2\pi\rho_0 = \frac{h}{m_e v_z} \tag{14}$$

$$v_z = \frac{h}{m_e 2\pi\rho_0} \tag{15}$$

$$v_z = \frac{\hbar}{m_e \rho_0} \tag{16}$$

Next we put in the appropriate numbers,

$$v_z = \frac{1.054 * 10^{-34} Js}{(9.1 * 10^{-31} kg)(0.597 * 5.29177 * 10^{-11} m)} \tag{17}$$

$$v_z = 3.864 * 10^6 ms^{-1}. \tag{18}$$

Aside from the level of precision, this is the same value given under "Theoretical Velocity" in Table 35.1 for Peak #1.

Converting this velocity value to a kinetic energy using $K.E. = \frac{1}{2}m_e v_z^2$ and changing the units to electronvolts produces the value of 42.3 eV, also in agreement with GUT_CP Table 35.1.

1.5 WHY DO THE HYPERBOLIC ELECTRONS HAVE THE PARTICULAR RADII SHOWN IN TABLE 35.1?

Mills's argument for the stability of the Hyperbolic Electron of radius $r_0 = 0.597a_0$ uses equation 35.94 of GUT_CP, which is

$$\frac{\hbar^2}{m_e r^3} = \frac{e^2}{4\pi\epsilon_0 r^2} + \frac{\hbar^2}{2m_e r^3}\sqrt{s(s+1)} \tag{19}$$

According to page 1628/1832 this equation is to describe "force balance" between the centrifugal, electric, and magnetic forces in the Hyperbolic Electron.

's' is the Spin Quantum Number which we set equal to 1/2. Then we have,

$$\frac{\hbar^2}{m_e r^3} = \frac{e^2}{4\pi\epsilon_0 r^2} + \frac{\hbar^2}{m_e r^3}\frac{1}{2}\sqrt{\frac{3}{4}} \tag{20}$$

$$\frac{\hbar^2}{m_e r^3}\left(1 - \frac{1}{2}\sqrt{\frac{3}{4}}\right) = \frac{e^2}{4\pi\epsilon_0 r^2} \tag{21}$$

$$\left(\frac{4\pi\epsilon_0}{e^2}\right)\left(\frac{\hbar^2}{m_e}\right)\left(1 - \frac{1}{2}\sqrt{\frac{3}{4}}\right) = r \tag{22}$$

Next we use a formula for the Bohr Radius from standard Quantum Mechanics, found in Bransden/Joachain [3] p.27

$$a_0 = \frac{(4\pi\epsilon_0)\hbar^2}{me^2} \tag{23}$$

and we have

$$a_0\left(1 - \frac{1}{2}\sqrt{\frac{3}{4}}\right) = r \tag{24}$$

which is identical to equation 35.95 of GUT_CP.

1.6 WHERE DOES THE "FORCE BALANCE" EQUATION COME FROM?

We have already, in equation (19) above, quoted the Force Balance equation,

$$\frac{\hbar^2}{m_e r^3} = \frac{e^2}{4\pi\epsilon_0 r^2} + \frac{\hbar^2}{2m_e r^3}\sqrt{s(s+1)} \tag{25}$$

$$Centrifugal\ Force\ =\ Electric\ Force\ +\ Magnetic\ Force \tag{26}$$

ELECTRIC FORCE TERM

The Electric Force term is equal to what the Electric Force between two point charges of charge e at separation distance r would be according to Coulomb's Law

$$F_{Coulomb} = \frac{e^2}{4\pi\epsilon_0 r^2} \tag{27}$$

Supposing we have a spherical surface of total charge e and radius r_A. From Newtonian theory, we know that the field produced by this spherical surface of charge is equal to that of a point charge for $r > r_A$ and is zero for $r < r_A$.

What about the field acting on a point on the actual surface of the sphere itself? Suppose we consider a spherical surface S of charge e in an xyz-coordinate system, such that the spherical surface is centred at the origin and has radius 1.

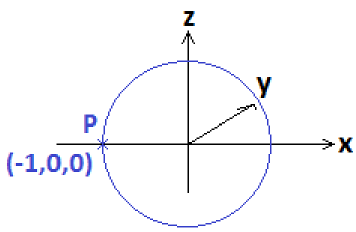

Figure 6: *Spherical Surface of charge* e *centred at origin.*

Consider a test point P at (-1,0,0). The field at P cannot be zero, because P is positioned such that every point of charge on S, apart from the charge at P itself, is to the positive-x side of P. Therefore, this charge will set up a field at P in the negative-x direction. It seems like a fair assumption to say that the field at P is equal to what it would be if the charge were all concentrated at the origin, that is,

$$E_P = \frac{1}{4\pi\epsilon_0}\frac{e}{r_A^2}. \tag{28}$$

At all points on the spherical surface, the magnitude of the field is the same as it is at P. The force, which is equal to field multiplied by charge, is

$$F = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_A^2}. \tag{29}$$

This is the effectively the "*self interaction*" force of the sphere acting upon itself. Equation (29) is the same as the Electric Force term in equation (25).

However, the Hyperbolic Electron is not actually a uniform-density spherical surface of charge, as shown by equations (6) and (7). So we can still ask the question as to why the Electric Force term in equation (25) gives the correct answer.

CENTRIFUGAL FORCE TERM

Actually, for motion of a particle around a circular path it is more accurate to refer to the force directed towards the centre as the *centripetal force* given by

$$F_c = \frac{mv^2}{r}. \tag{30}$$

We start with the de Broglie relation

$$p = \frac{h}{\lambda} \tag{31}$$

We set the wavelength equal to $2\pi r_n$, in which r_n is the radius of the Charge Circle. Then,

$$mv = \frac{h}{2\pi r_n} \tag{32}$$

$$m^2v^2 = \frac{\hbar^2}{r_n^2} \tag{33}$$

$$\frac{mv^2}{r} = \frac{\hbar^2}{mr_n^3} \tag{34}$$

Thus, in equation (34) we have found the Centripetal Force Term of equation (25). However there are still questions we can ask, such as:

The radius r in equation (25) is the radius of the Hyperbolic Electron. However, the Filamentary Circles in the Hyperbolic Electron are not all of this radius, rather, they become progressively smaller as one moves towards the poles. So why then does equation (25) give the correct answer?

MAGNETIC FORCE TERM

On page 1627/1832 Mills refers to GUT_CP equation 7.31 as the origin of the Magnetic Force term. We go to page 285/1832 and we find Equation 7.31 stated as

$$\mathbf{F}_{mag} = \frac{1}{4\pi r_2^2}\nabla\left(\Delta E_{mag}^{spin}\right) = \frac{1}{4\pi r_2^2}\frac{\delta E_{mag}^{spin}}{\delta r}\mathbf{i}_r = -\frac{1}{4\pi r_2^2}\frac{\hbar^2}{Zm_e r^3}\sqrt{\frac{3}{4}}\mathbf{i}_r \tag{35}$$

We are interested in the rightmost form of this equation. The $1/(4\pi r_2^2)$ factor converts from force to force per unit area, and we leave it out. Then we set $Z = 1$ and we get

$$\mathbf{F}_{mag} = -\frac{\hbar^2}{m_e r^3}\sqrt{\frac{3}{4}}\mathbf{i}_r \tag{36}$$

This is identical to the Magnetic Force term in equation (25) with $s = 1/2$ apart from the sign and a factor of 2.

Next, we use the Biot Savart Law and $F = q\mathbf{v} \times \mathbf{B}$ to see if we can obtain results similar to those of Mills in GUT_CP Chapter 7.

Consider first the Orbitsphere of an electron bound around an atomic nucleus at distance r from the nucleus. Then, consider that we bring in a second Orbitsphere also of radius r, and superpose it upon the first Orbitsphere such that the Great Circles of flowing current composing each Orbitsphere all superpose, but those of the second orbitsphere are of opposite directions to those of the first Orbitsphere.

Next, we consider a simplified model of the two Orbitspheres, in which the Orbitspheres have been reduced to point particles of charge e initially on opposite sides of a circle of radius r from the nucleus. The point particles are moving opposite directions along the circle (Figure 7).

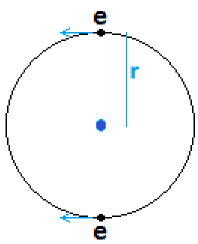

Figure 7: *Simplified model in which two Orbitspheres have been reduced to point particles moving around a single Great Circle.*

We apply the Biot Savart Law,

$$\mathbf{B} = \frac{\mu_0}{4\pi} q \frac{\mathbf{v} \times \mathbf{r}}{r^3} \tag{37}$$

We then obtain the field at the second point-particle electron due to the first point-particle electron as

$$B = \frac{\mu_0}{4\pi} e \frac{v_1}{(2r)^2} \tag{38}$$

This field is perpendicular to the plane of the circle. Then, we use $F = q\mathbf{v} \times \mathbf{B}$ to get the force on the second point-particle electron as

$$F = \frac{\mu_0}{4\pi}\frac{e^2 v_1^2}{(2r)^2} \tag{39}$$

This force is directed along the line segment joining the two particles, and is towards the nucleus. Also, the first point-particle electron experiences a force of equal magnitude towards the nucleus, due to the field set up by the second point-particle electron.

Then, we use the formula for the velocity, derived by using the de Broglie relation and setting the circumference of the circle equal to the de Broglie wavelength

$$v = \frac{\hbar}{m_e r_n}. \tag{40}$$

Combining equations (39) and (40), we obtain

$$F = \frac{\mu_0}{4\pi}\frac{e^2}{(2r)^2}\frac{\hbar^2}{(m_e r_n)^2} \tag{41}$$

This can be compared with equation 7.14 on page 283/1832 of GUT_CP, which is

$$\mathbf{F}_{mag} = -\frac{1}{4\pi r_2^2}\frac{\mu_0 e^2 \hbar^2}{2r_1 m_e^2 r_2^2}\sqrt{\frac{3}{4}}\mathbf{i}_r \tag{42}$$

Neglecting the *1/surface area* prefactor, we see that equations (41) and (42) agree apart from a constant numerical factor. For equation (41) this numerical factor is

$$\frac{1}{4\pi} \times \frac{1}{4} \tag{43}$$

and for equation (42) the numerical factor is, neglecting the *1/surface area* prefactor,

$$\frac{1}{2} \times \sqrt{\frac{3}{4}} \tag{44}$$

Further study is required to understand in greater detail the various terms of the Force Balance Equation as applied to Hyperbolic Electrons.

2 APPENDIX A:

VERIFICATION THAT MASS DENSITY FORMULA FOR FREE ELECTRON YIELDS TOTAL MASS EQUAL TO ELECTRON MASS

We have,

$$\sigma_m(\rho) = \frac{m_e}{\frac{2}{3}\pi\rho_0^3}\sqrt{\rho_0^2 - \rho^2} = A\sqrt{\rho_0^2 - \rho^2}. \tag{45}$$

Then, we want to find

$$m_{total} = \int_0^{\rho_0} \sigma_m(\rho) \times 2\pi\rho d\rho \tag{46}$$

Proceeding to do the integration,

$$\begin{aligned} m_{total} =& \int_0^{\rho_0} A \cdot 2\pi\rho\sqrt{\rho_0^2 - \rho^2}d\rho \\ =& \left[\frac{2}{3} \cdot [\rho_0^2 - \rho^2]^{3/2} \cdot (-1) \cdot A\pi\right]_0^{\rho_0} \\ =& \frac{2}{3}A\pi(-1)\left[[\rho_0^2 - \rho^2]^{3/2}\right]_0^{\rho_0} \\ =& \frac{2}{3}A\pi(-1)\left\{[\rho_0^2 - \rho_0^2]^{3/2} - [\rho_0^2 - 0^2]^{3/2}\right\} \\ =& \frac{2}{3}A\pi(-1)\left\{0 - \rho_0^3\right\} \\ =& \frac{2}{3}A\pi\rho_0^3 \\ =& \frac{2}{3}\pi\rho_0^3 \cdot A \\ =& \frac{2}{3}\pi\rho_0^3 \cdot \frac{m_e}{\frac{2}{3}\pi\rho_0^3} \\ =& m_e \end{aligned} \tag{47}$$

ACKNOWLEDGEMENTS

The author thanks Hugues Vermeiren for providing online an example of a geometric diagram made using the extension to Latex known as the *tikz* package. This example and source code served as a *tikz* tutorial to the author. The example and code can be found at

http://www.texample.net/tikz/examples/
"Plane Sections of the Cylinder - Dandelin Spheres"

This report was created using MikTeX and material from the LaTeX Wikibook. The LaTeX Wikibook is released under the Creative Commons License.

References

[1] R. Mills, "The Grand Unified Theory of Classical Physics." available online at www.blacklightpower.com.

[2] B. Jones, "The Orbitsphere - Alternative Presentation." available online at www.orbitsphere-alternative-presentation.com.

[3] B. H. Bransden and C. J. Joachain, *Quantum Mechanics.* Prentice-Hall, 2nd ed., 2000.

[4] T. Tantau, "TikZ & PGF - Manual for version 2.10." available online at http://mirror.ox.ac.uk/sites/ctan.org/graphics/pgf/base/doc/generic/pgf/pgfmanual.pdf.

[5] F. C. Witteborn and W. M. Fairbank, "Experimental comparison of the gravitational force on freely falling electrons and metallic electrons," *Phys. Rev. Lett.*, vol. 19, pp. 1049–1052, Oct 1967.

[6] J. Gribbin *New Scientist*, p. 15, January 25th 1997.

[7] "Wikipedia entries on 'Hyperboloid', 'Tractrix', 'Pseudosphere', and 'Gaussian Curvature'." Available on the Web at http://en.wikipedia.org.