

$$\begin{aligned} \text{length}^2 &= \delta x^2 + \delta y^2 \\ \text{length} &= \sqrt{\delta x^2 + \delta y^2} \end{aligned}$$

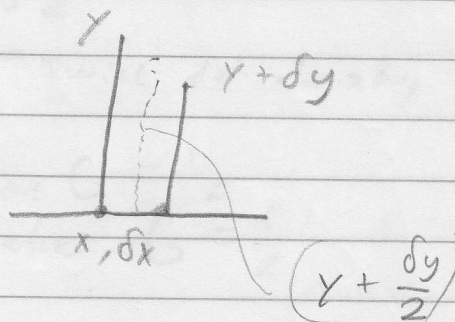
$$y = \sqrt{1-x^2}$$

$$y = (1-x^2)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot \frac{d}{dx} (1-x^2)$$

$$\frac{dy}{dx} = \frac{1}{2} (1-x^2)^{-\frac{1}{2}} \cdot -2x$$

$$\frac{dy}{dx} = (1-x^2)^{-\frac{1}{2}} \cdot -x$$



Area

$$= 2\pi \left(y + \frac{\delta y}{2} \right) \cdot \sqrt{\delta x^2 + \delta y^2}$$

$$= 2\pi \left(y + \frac{\delta y}{2} \right) \sqrt{1 + \left(\frac{\delta y}{\delta x} \right)^2} \cdot \delta x$$

$$= 2\pi \cdot y \cdot \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \cdot dx$$

$$2\pi \sqrt{1-x^2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \cdot dx$$

$$\frac{dy}{dx} = -x (1-x^2)^{-\frac{1}{2}}$$

$$\left(\frac{dy}{dx} \right)^2 = x^2 (1-x^2)^{-1} = \frac{x^2}{1-x^2}$$

$$\rightarrow 2\pi \sqrt{1-x^2} \sqrt{1 + \frac{x^2}{1-x^2}} dx$$

$$2\pi \sqrt{1-x^2} \sqrt{\frac{1-x^2}{1-x^2} + \frac{x^2}{1-x^2}} dx$$

$$2\pi \sqrt{1-x^2} \sqrt{\frac{1}{1-x^2}} dx$$

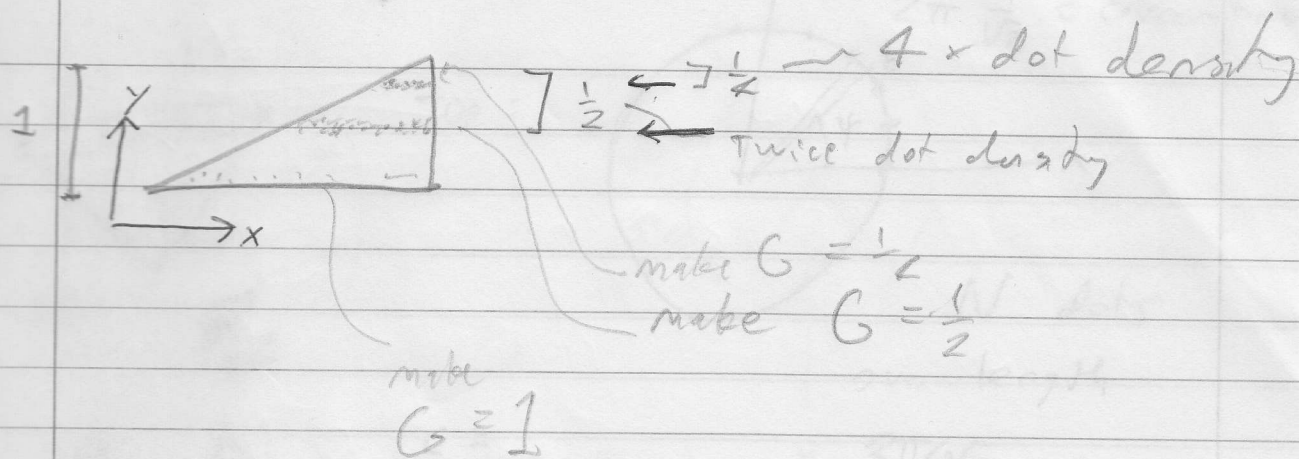
$$2\pi$$

$$\int 2\pi dx = \boxed{2\pi x}$$

$$2\pi y \cdot \sqrt{1 + \left(\frac{x^2}{y^2} \right)} dx$$

$$2\pi y \cdot \sqrt{\frac{y^2 + x^2}{y^2}} dx$$

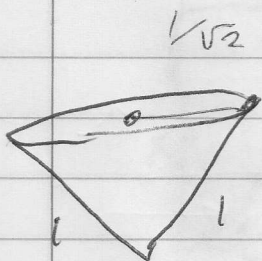
$$2\pi$$



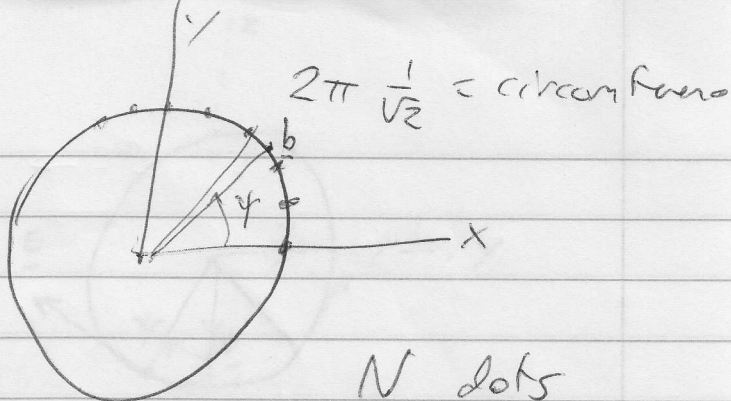
In above triangle, looking at density of dots along narrow bands parallel to x -axis

density of dots $\times G(y) = \text{CONSTANT}$

so density of dots $\times 2 \Rightarrow G \times \frac{1}{2}$



TOP :



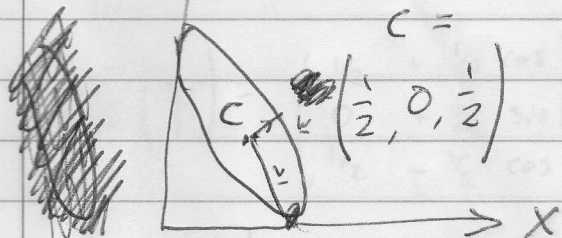
N dots

over length

$$2\pi/\sqrt{2}$$

$$\therefore \text{dots per unit length} = \frac{N}{2\pi/\sqrt{2}}$$

$$\frac{\text{dots in } \Delta L}{\Delta L} = \frac{N}{\pi\sqrt{2}}$$



$$\underline{v} = \begin{pmatrix} \frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{pmatrix} \quad \underline{w} = \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

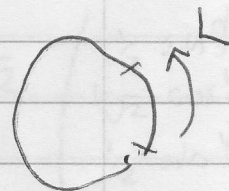
corresponding to ψ

$$\underline{b}(\psi) = \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix} + \underline{v} \cos \psi + \underline{w} \sin \psi$$

$$= \begin{pmatrix} 1/2 \\ 0 \\ 1/2 \end{pmatrix} + \cos \psi \cdot \begin{pmatrix} 1/2 \\ 0 \\ -1/2 \end{pmatrix} + \sin \psi \cdot \begin{pmatrix} 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix}$$

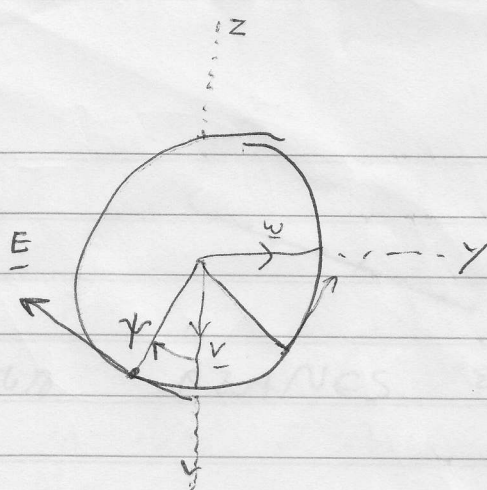
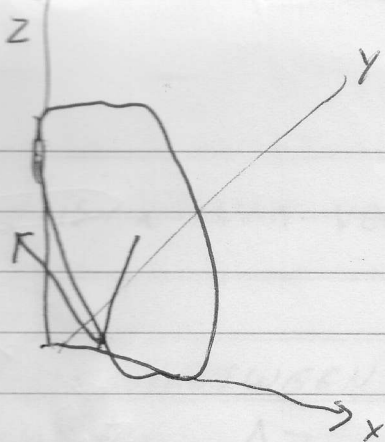
$$\underline{b}|_z = \frac{1}{2} - \frac{1}{2} \cos \psi$$

$$\text{dots per } \Delta z = \frac{\text{dots in } \Delta L}{\Delta L} \cdot \frac{\Delta L}{\Delta z}$$



$$\text{dots per } \Delta z = \frac{N}{2\pi/\sqrt{2}} \cdot \frac{\Delta L}{\Delta z}$$

$$\begin{aligned} & \text{at } \begin{pmatrix} 1/2 \\ -1/\sqrt{2} \\ 1/2 \end{pmatrix} \\ & \frac{\Delta L}{\Delta z} = \sqrt{2} \\ & \frac{N}{2\pi/\sqrt{2}} \cdot \sqrt{2} = \frac{N}{2\pi} \cdot 2 = \frac{N}{\pi} \end{aligned}$$



$$\underline{b}(\psi) = \begin{pmatrix} \frac{1}{2} + \frac{1}{2} \cos \psi \\ 0 + \frac{1}{\sqrt{2}} \sin \psi \\ \frac{1}{2} - \frac{1}{2} \cos \psi \end{pmatrix}$$

$$\frac{d\underline{b}(\psi)}{d\psi} = \text{No. should differentiate with respect to length}$$

$$\begin{aligned} \text{length of arc} &= r\theta \\ &= \frac{1}{\sqrt{2}} \cdot \psi \end{aligned}$$

Circle, radius r , angle θ
length of arc
traversed = $r\theta$

$$\frac{d\underline{b}}{d(\frac{1}{\sqrt{2}}\psi)} = \sqrt{2} \frac{d\underline{b}}{d\psi}$$

$$\frac{d\underline{b}}{d\psi} = \begin{pmatrix} -\frac{1}{2} \sin \psi \\ \frac{1}{\sqrt{2}} \cos \psi \\ \frac{1}{2} \sin \psi \end{pmatrix}$$

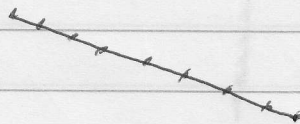
$$\frac{d\underline{b}}{d\ell} = \sqrt{2} \begin{pmatrix} -\frac{1}{2} \sin \psi \\ \frac{1}{\sqrt{2}} \cos \psi \\ \frac{1}{2} \sin \psi \end{pmatrix}$$

$$\begin{aligned} \left| \frac{d\underline{b}}{d\psi} \right|^2 &= \frac{1}{4} \sin^2 \psi + \frac{1}{2} \cos^2 \psi + \frac{1}{4} \sin^2 \psi \\ &= \frac{1}{2} (\sin^2 + \cos^2) \\ &= \frac{1}{2} \end{aligned}$$

$$\left| \frac{d\underline{b}}{d\psi} \right| = \frac{1}{\sqrt{2}}$$

$$\left| \frac{d\underline{b}}{d\ell} \right| = 1$$

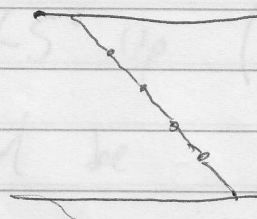
$\frac{db}{dl}$ IS A UNIT VECTOR



FOR # BETWEEN TWO PLANES z, z_2
separation Δz

$$\frac{\Delta z}{\left. \frac{db}{dl} \right|_z} = \text{number of unit vectors in this space}$$

$$\frac{\Delta z}{\frac{\sqrt{2}}{2} \sin \psi}$$



$\frac{N}{\pi \sqrt{2}} = \text{angular momentum lobes per unit length on the circle}$

$$\frac{\Delta z}{\frac{\sqrt{2}}{2} \sin \psi} \frac{N}{\pi \sqrt{2}} = \text{number of lobes } \psi, \Delta z$$

$$= \frac{\Delta z}{\sin \psi} \frac{N}{\pi}$$

$$\frac{1}{2} - \frac{1}{2} \cos \psi = z$$

$$= \frac{\Delta z N_{4\pi}}{\sqrt{1 - \cos^2 \psi}}$$

$$1 - \cos \psi = 2z$$

$$-\cos \psi = 2z - 1$$

$$\cos \psi = 1 - 2z$$

$$\cos^2 \psi = 1 - 4z + 4z^2$$

$$= \frac{\Delta z N_{4\pi}}{\sqrt{4z - 4z^2}}$$

$$= \frac{\Delta z N / \pi}{2 \sqrt{z - z^2}}$$

$$\text{when } z = \frac{1}{2} :$$

$$= \frac{\Delta z N / \pi}{2 \sqrt{\frac{1}{4}}} = \Delta z \frac{N}{\pi}$$

$\frac{N}{\pi\sqrt{2}}$ = number of dots per unit length on circle.



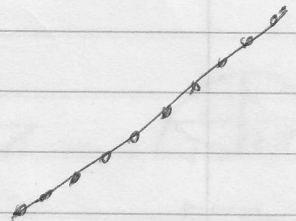
IF we stretch circle into a line going straight up along z -axis
number of dots in Δz

is $\Delta z \cdot \frac{N}{\pi\sqrt{2}}$

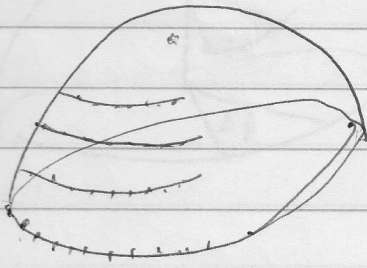
And if ~~this~~ this line is 45° up. (45° to z -axis,
number of dots in Δz should be

$$\Delta z \cdot \frac{N}{\pi\sqrt{2}} \times \sqrt{2}$$

$$= \Delta z \cdot \frac{N}{\pi}$$



CONFIRMS RESULT



at base of CONE DOME,

$$\text{RADIUS} = 1$$

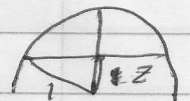
$$\text{CIRCUMFERENCE} = 2\pi$$

$$\text{NUMBER OF POINTS} = M \quad \frac{M}{2\pi} = \text{points per unit length}$$

FOR EACH LATITUDINAL CIRCLE
NUMBER OF POINTS = M

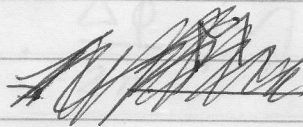
FOR LATITUDINAL CIRCLE, height z

$$2\pi \sqrt{1-z^2} = \text{circumference}$$

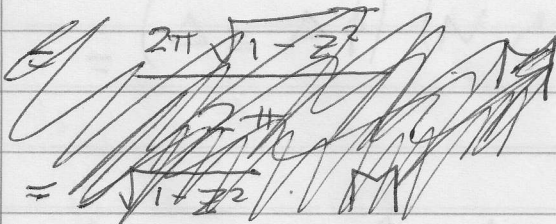


$$\sqrt{1-z^2} = \text{radius}$$

Number of points
per unit length around
latitudinal circle



$$= \frac{M}{2\pi \sqrt{1-z^2}}$$



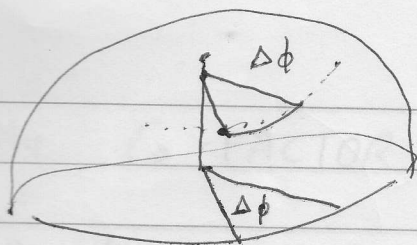
IN
LONGITUDINAL
REGION, $\Delta\phi$ THE

Number of planes on
CONE DOME,

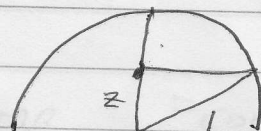
from height z to $z + \Delta z$

$$= \frac{\Delta z \cdot N/\pi}{2 \sqrt{1-z^2}}$$





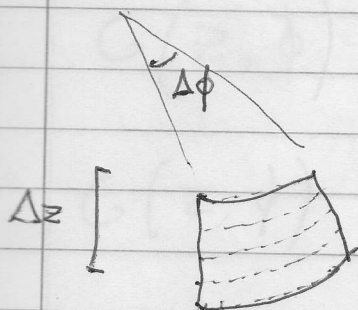
Arc length



$$\text{Arc length} = \sqrt{1-z^2} \cdot \Delta\phi$$

$$\text{Number of points} = \sqrt{1-z^2} \cdot \Delta\phi \cdot \frac{M}{2\pi\sqrt{1-z^2}}$$

$$= \frac{\Delta\phi}{2\pi} M$$



region, from z to $z + \Delta z$,
 $\Delta\phi$

number of points =

number of planes $\times$ number of points in each plane

$$= \frac{\Delta z \cdot N/4\pi}{2\sqrt{z-z^2}} \cdot \frac{\Delta\phi}{2\pi} M$$

$$= (\Delta z \Delta\phi) (N M) \left(\frac{1}{4\pi^2 \sqrt{z-z^2}} \right)$$

$$\text{surface area} = 2\pi \cdot \Delta z \cdot \frac{\Delta\phi}{2\pi} = \Delta z \Delta\phi$$

But does this REALLY lead to a uniform charge density, as Dr. Mills states is shown numerically in

Chapter 2 of GUT-CD

the G-FACTOR:

$$\text{Density of Points} \times G \text{ FACTOR} = \text{const} = \alpha$$

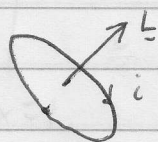
$$G = \frac{\alpha}{\text{Density of Points}}$$

$$G(z, \phi) = \frac{\alpha}{NM \frac{1}{4\pi^2} \frac{1}{\sqrt{z-z^2}}}$$

$$G(z, \phi) = \alpha' \sqrt{z-z^2}$$

the G FACTOR MULTIPLIES

the ANGULAR MOMENTUM associated
with the corresponding Great Circle



Hence, if Great Circle
has charge q of mass m
making current i (~~q~~ i measured
in e.g. nanoamperes)

$$\text{then } q \Rightarrow Gq$$

$$m \Rightarrow Gm$$

$$\text{and } i \Rightarrow Gi$$

$$\text{so } L \Rightarrow GL$$

But does this REALLY lead to
a uniform charge density, as Dr. Mills
states ~~in~~ is shown numerically in
~~pp 88-89~~ Chapter 1 of GUT-CP

$$y = \sqrt{x - x^2}$$

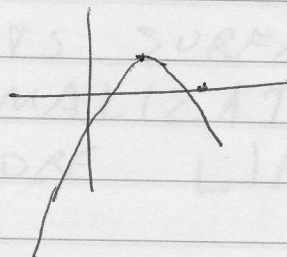
$$\bar{y} = x - x^2 \quad \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$= -x^2 + x$$

$$= -(x^2 - x)$$

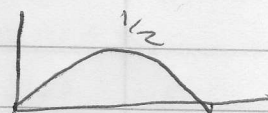
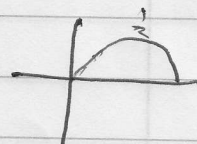
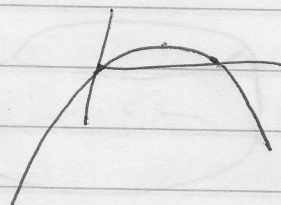
$$y = -\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}\right]$$

$$= -\left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$$



$$0 = -(x^2 - x)$$

$$\bar{y} = x(1-x)$$

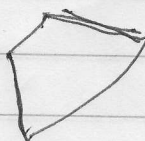


$$\frac{d\bar{y}}{dx} = 1 - 2x$$

$$\left. \frac{dy}{dx} \right|_{x=0} = 1$$

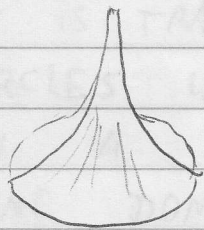
$$x=0$$

$$\left. \frac{dy}{dx} \right|_{x=1} = -1$$

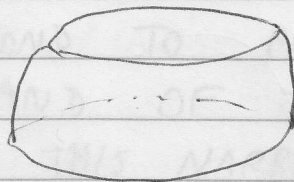


Question:
If density of the
planes on the
cone dome were
constant, what then
would we have?

IN ANSWER TO QUESTION
POSED EARLIER, WHICH WAS
"DOES THE CONE DOME, WHEN EACH
VECTOR ENDING ON ITS SURFACE IS
MULTIPLIED BY NORMALIZATION
FACTOR BECOME MORE LIKE

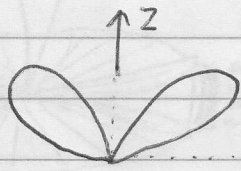


OR



?

ACTUALLY IT BECOMES
MORE LIKE THIS



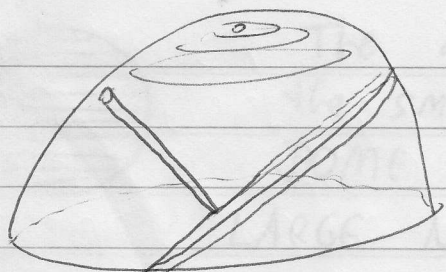
THE PETALS, RESIDUE
ROTATED

ABOUT $+Z$ AXIS.

THE "PETAL CUP"

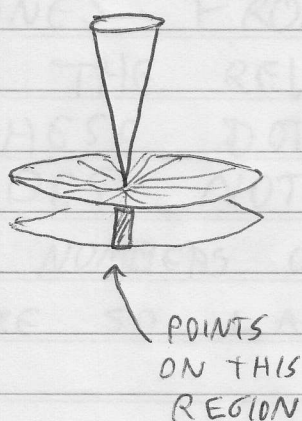
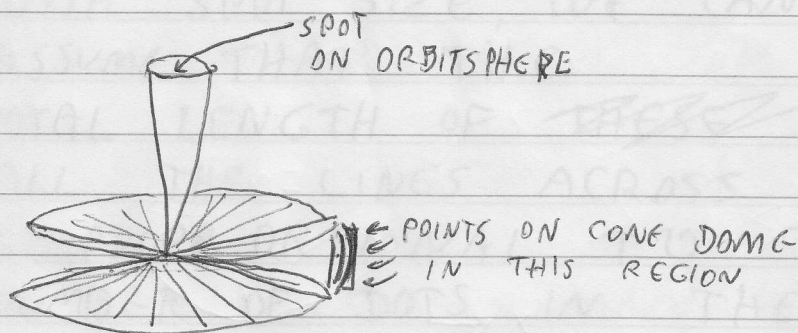
ORBITSPHERE

& CONE DOME
SUPERPOSED



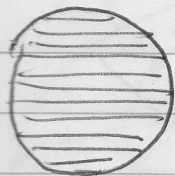
CIRCULAR

CONSIDER A SMALL SPOT ON THE ORBITSPHERE
IT IS TRAVERSED BY PIECES OF GREAT
CIRCLES WHICH CORRESPOND TO POINTS
ON A NARROW BAND OF THE
CONE DOME. CALL THIS NARROW BAND
THE "HANDLE"



Cause
lines
across spot
like this



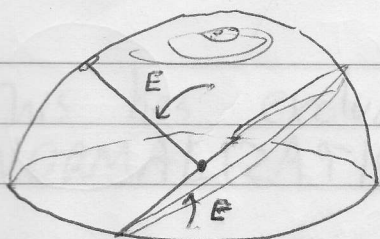


The density of points on the small region of the CONE DOME is such that a VERY LARGE NUMBER OF LINES are DRAWN ACROSS THE SPOT.

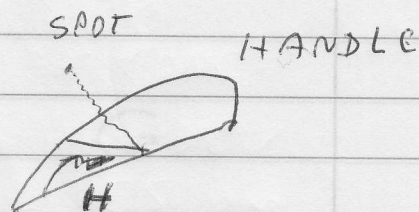
~~BECAUSE~~ THIS IS EVEN THOUGH THE SPOT ITSELF IS SMALL.

SINCE THE NUMBERS OF LINES ACROSS THE SPOT ARE LARGE SO THAT LINE SEPARATION IS SMALL COMPARED WITH SPOT SIZE, WE CAN ASSUME THAT THE TOTAL LENGTH OF ~~THESE CROSSING LINES~~ ALL THE LINES ACROSS THE SPOT IS PROPORTIONAL TO THE NUMBER OF DOTS IN THE SMALL SECTION OF THE CONE DOME THAT WE ARE DRAWING THESE CROSS LINES FROM.

THE RELATIVE ORIENTATION OF THESE DOTS IN THE CONE DOME DOES NOT MATTER, SINCE THE NUMBERS OF DOTS AND CORRESPONDING ARE SO LARGE



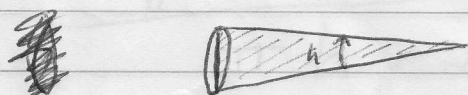
radius = 1



Assume the NARROW BAND is so narrow that we can treat it like a piece of cylinder, profile radius = 1
height h



h is determined by the spot size



h radians =
angle (NOT SOLID ANGLE)
subtended by spot size

TOTAL CHARGE IN THE SPOT = Q_{TOTAL}

$$Q_{TOTAL} \propto \int_{H=0}^{H=\pi} \text{density of points on cone dome} \cdot h \cdot dH$$

FROM EARLIER, density of points on Cone Dome = $NM \frac{1}{4\pi^2 \sqrt{z-z^2}}$

$$= \frac{\gamma}{\sqrt{z-z^2}}$$

$$Q_{TOTAL} \propto \int_{H=0}^{H=\pi} \frac{\gamma}{\sqrt{z-z^2}} \cdot h \cdot dH$$

$\gamma = \text{constant}$

~~FROM~~

$$z = \sin H \cdot \sin E$$

This has excluded the
NORMALIZATION FACTOR, G

FROM EARLIER, $G = \alpha' \sqrt{z - z^2}$

if we multiply DOT DENSITY by G
we get

$$Q \propto \int_0^\pi \frac{\gamma}{\sqrt{z - z^2}} h \alpha' \sqrt{z - z^2} dH$$

$$= \int_0^\pi \gamma \alpha' h dH$$

$$\boxed{Q = \pi h \gamma \alpha'}$$

= CONSTANT WITH RESPECT
TO SPOT LOCATION POLAR
ANGLE θ .